International Journal of Theoretical Physics, Vol. 37, No. 3, 1998

Constant-Cutoff Approach to Axially Symmetric Dibaryons

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Received October 6, 1997

We suggest a quantum stabilization method for the $SU(2)$ σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.*, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. We then study the dibaryon configurations in this approach, using the generalized axially symmetric ansatz to determine the soliton background. Thus we calculate the rotational contributions to the masses of the axially symmetric dibaryons and show that they are in qualitative agreement with the results obtained using the complete Skyrme model. We conclude also that, as in the case of the complete Skyrme model, the lowest allowed $S = -2$ state has the quantum numbers of the H-particle. We find that in the present approach, similarly to the case of the complete Skyrme model, this particle is bound, even though the neglected vacuum effects might contribute to the unbinding of the H-particle.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \text{Tr}\, \partial_\mu U\, \partial^\mu U^+ \tag{1.1}$$

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0020-7748/98/0300-105 1$15.00/0

$$U = \frac{2}{F_\pi}(\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2), $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion-isotriplet.

The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1)

$$\mathscr{L}_{\text{Sk}} = \frac{1}{32e^2}\operatorname{Tr}[U^+\partial_\mu U, U^+\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It has been shown by several authors (Adkins *et al.*, 1983; see also Witten, 1979, 1983a,b; for extensive references see Holzwarth and Schwesinger, 1986, and Nyman and Riska, 1990) that, after the collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\,\boldsymbol{\tau}\cdot\mathbf{r}_0\,F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand, it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated therefore a possibility to build a stable single-baryon ($n = 1$) quantum state in the simple chiral theory, with the Skyrme stabilizing term (1.3) omitted. MW showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r},t) = A(t)U_0(\mathbf{r})\,A^+(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4}F_\pi^2\frac{1}{\chi''(0)}a + \frac{1}{2}\frac{[\chi''(0)]^3}{(\pi/4)F_\pi^2 b}J(J+1) \tag{1.5}$$

where

$$a = \int_0^\infty \left[\frac{1}{4} s^2 \left(\frac{d\mathcal{F}}{ds}\right)^2 + 8 \sin^2\left(\frac{1}{4}\mathcal{F}\right)\right] ds \tag{1.6}$$

$$b = \int_0^\infty ds \, \frac{64}{3} s^2 \sin^2\left(\frac{1}{4}\mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \tfrac{1}{4}\mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2}\left(\frac{\pi}{4}\right)^2 \frac{a^3}{b} J(J+1)\right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-model, it is possible, after collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by Mignaco and Wulck (1989) is not correct since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m \text{ even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy}\cos\left[\frac{\sqrt{7}}{2}\ln(cy) + \alpha\right], \qquad m \text{ odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen appropriately. When $F(0) = -n\pi$, then we want to know which of these two solutions is approached by $F(y)$ when $y \to 0$ ($r \to \infty$). In order to answer that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2\,dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2\,dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ ($r \to 0$) is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\,dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to \text{const}$ as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a,b, 1992), I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\tau\cdot\mathbf{r}_0F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. We then study the dibaryon configurations in this approach, using the generalized axially symmetric ansatz to determine the soliton background. Thus we calculate the rotational contributions to the masses of the axially symmetric dibaryons and show that they are in qualitative agreement with results obtained in the complete Skyrme model (Thomas *et al.*, 1994). We conclude also that, as in the complete Skyrme model (Thomas *et al.*, 1994), the lowest allowed $S = -2$ state has the quantum numbers of the H-particle. We find that in the present approach, similarly to the case of the complete Skyrme model (Thomas *et al.*, 1994), this particle is bound, even though the neglected vacuum effects might contribute to the unbinding of the H-particle.

The reason why the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff ε is introduced, then the boundary conditions $F(\varepsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. In Balakrishna *et al.* (1991) an interesting analogy with the damped pendulum is discussed, showing clearly that as long as $\varepsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\varepsilon \to 0$ the chiral phase diverges at the lower limit.

2. CONSTANT-CUTOFF STABILIZATION

Substituting (1.4) into (1.1), we obtain for the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\varepsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\varepsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\varepsilon, \infty]$. The cutoff itself is introduced following as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr} \left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\varepsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \varepsilon(t)]$ now depends implicitly on time t through $\varepsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U\, \partial^\mu U^+)\, d^3\mathbf{r} \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t) U_0(\mathbf{r}, t) A^+(t), \qquad U^+(\mathbf{r}, t) = A(t) U_0^+(\mathbf{r}, t) A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\tau \cdot \mathbf{r}\, F[r; \varepsilon(t)]\} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+)\, d^3\mathbf{r} = -E_0 \tag{2.6}$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and is equal to

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U\, \partial_0 U^+)\, d^3\mathbf{r} = bx^2\, \mathrm{Tr}[\partial_0 A\, \partial_0 A^+] + c[\dot{x}(t)]^2 \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F\, y^2 dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2 dy \tag{2.8}$$

with $x(t) = [\varepsilon(t)]^{3/2}$ and $y = r/\varepsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \; a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2 \left(\frac{dF}{dy}\right)^2 + 2 \sin^2 F \right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2\, \dot{\alpha}_\nu \dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A\, \partial_0 A^+) = 2\dot{\alpha}_\nu \dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \alpha^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \tag{2.11}$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3} \frac{ab}{J(J+1)} \right]^{-3/8} \Rightarrow \varepsilon^{-1} = \left[\frac{2}{3} \frac{ab}{J(J+1)} \right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3} \left[\frac{3}{2} \frac{a^3}{b} J(J+1) \right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$ and $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{3/2}F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as found by Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ-resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2, giving exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = \frac{1}{2}$) the cutoffs are equal to

$$\varepsilon = \begin{cases} 0.22 \text{ fm} & \text{for} \quad F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} & \text{for} \quad F_\pi = 150 \text{ MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. THE SU(3)-EXTENDED SIMPLIFIED SKYRME MODEL

3.1. Introduction

It was first proposed in Jaffe (1977), based on a bag-model calculation, that some hexa-quark states may be stable against strong decays. However, it has been shown (Aeerts *et al.*, 1978; Liu and Wong, 1982; Mulders and Thomas, 1983) that symmetry-breaking effects, center-of-mass corrections, pion cloud around the bag, etc., tend to decrease the binding of the hexaquark states and to increase the uncertainty of their existence. Although the analysis so far provides no evidence for a stable H-dibaryon, new experiments are being carried out to further investigate the issue (Quinn, 1992).

It is therefore of interest to apply the constant-cutoff approach described above in the CHK model of strange axially symmetric dibaryons (Callan and Klebanov, 1985; Callan *et al.*, 1988) to study the strange dibaryon stability and compare the corresponding results obtained using the complete Skyrme model (Thomas *et al.*, 1994).

3.2. The Effective Interaction

The Lagrangian density for a dibaryon system with pseudoscalar mesons is given, with Skyrme stabilizing term omitted, by (Dalarsson, 1993, 1995a–d, 1996a–c, 1997a–d)

$$\begin{aligned}\mathcal{L} = {} & \frac{F_\pi^2}{16}\,\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^+ + \frac{F_\pi^2 m_\pi^2 + 2F_K^2 m_K^2}{48}\,\mathrm{Tr}(U + U^+ - 2) \\ & - \frac{1}{48}(F_K^2 - F_\pi^2)\,\mathrm{Tr}[(1 - \sqrt{3}\lambda_8)(U\,\partial_\mu U^+\,\partial^\mu U + U^+\,\partial_\mu U\,\partial^\mu U^+)] \\ & + \frac{1}{24}(F_K^2 m_K^2 - F_\pi^2 m_\pi^2)\,\mathrm{Tr}[\sqrt{3}\lambda_8(U + U^+)] \end{aligned} \tag{3.1}$$

where m_π and m_K are pion and kaon masses, respectively, and F_K is the kaon weak-decay constant with the empirical value $F_K = 226$ MeV. The first term in (3.1) is the usual σ-model Lagrangian, while the remaining three terms are all chiral- and flavor-symmetry-breaking terms, present in the mesonic sector of the model, which will be used in this form even for the multibaryon ($n > 1$) states. All flavor-symmetry-breaking terms in the effective Lagrangian (3.1) also break the chiral symmetry, just as quark-mass terms do in the underlying QCD Lagrangian. In addition to the action, obtained using the Lagrangian (3.1), the Wess–Zumino action in the form

$$\begin{aligned} S = {} & -\frac{iN_c}{240\pi^2} \\ & \times \int d^5x\, e^{\mu\nu\alpha\beta\gamma}\,\mathrm{Tr}[U^+\partial_\mu U\; U^+\,\partial_\nu U\; U^+\partial_\alpha U\; U^+\partial_\beta U\; U^+\partial_\gamma U] \end{aligned} \tag{3.2}$$

must be included into the total action of a dibaryon system, where N_c is the number of colors in the underlying QCD. The Wess–Zumino action defines the topological properties of the model, important for the quantization of the solitons. In the $SU(2)$ case the Wess–Zumino action vanishes identically and was therefore not present in the discussions of Sections 1 and 2 here.

In the present approach the meson–soliton field is written in the form

$$U = \sqrt{U_\pi}U_K\sqrt{U_\pi} \tag{3.3}$$

where U_π is $SU(3)$ extension of the usual $SU(2)$ skyrmion field used to describe the nucleon spectrum, and U_K is the field describing the kaons

$$U_\pi = \begin{bmatrix} u_\pi & 0 \\ 0 & 1 \end{bmatrix}, \qquad U_K = \exp\left\{ i\,\frac{2^{3/2}}{F_\pi}\begin{bmatrix} 0 & K \\ K^+ & 0 \end{bmatrix}\right\} \tag{3.4}$$

In the single-baryon ($n = 1$) sector the lowest energy states have the hedgehog structure within $SU(2)$ given by (1.4). The lowest dibaryon states (Dalarsson, 1993, 1995a–d, 1996a–c, 1997a–c) are characterized by an axially symmetric form of U leading to a torus-shaped baryon-number density, i.e., we have the $SU(2)$ ansatz

$$u_\pi(\mathbf{r}) = \exp[i\tau\cdot\eta F(r)], \qquad \eta = \begin{bmatrix} \sin\alpha\cos n\phi \\ \sin\alpha\sin n\phi \\ \cos\alpha \end{bmatrix} \tag{3.5}$$

where the variational function

$$\alpha = \theta + \sum_{k=1}^{m} a_k \sin(2k\theta) \tag{3.6}$$

is used instead of the usual spherical coordinate θ. The two-dimensional vector K in (3.4) is the kaon doublet

$$K = \begin{bmatrix} K^+ \\ K^0 \end{bmatrix}, \qquad K^+ = [K^- \; \overline{K}^0] \tag{3.7}$$

For $n \neq 1$ it is customary (Dalarsson, 1993, 1995a–d, 1996a–c, 1997a–c; Thomas *et al.*, 1994) to use the following ansatz for the kaon field:

$$K(\mathbf{r}, t) = k(r, t)\;\tau\cdot\eta\chi \tag{3.8}$$

where χ is a two-component spinor.

We now substitute (3.3), with U_π and U_K defined by (3.4), using (3.5) with (3.6), and (3.7) with (3.8), into the total action of the kaon–soliton system and expand U_K to second order in kaon fields (3.8), to obtain the effective interaction Lagrangian for the kaon–soliton system:

$$L = \int_{\varepsilon}^{\infty} r^2 dr \left[\dot{k}^+ \dot{k} - \frac{\partial k^+}{\partial r}\frac{\partial k}{\partial r} + i\lambda(r)(\dot{k}^+ k - k^+ \dot{k}) - (m_K^2 - v_0(r))k^+ k \right] \tag{3.9}$$

where ε is the constant cutoff defined as in (2.1) and we introduced the quantities $\lambda(r)$ and $v_0(r)$ as follows:

$$\lambda(r) = -\frac{N_c}{2\pi^2 F_K^2}\frac{\sin^2 F}{r^2}\frac{dF}{dr}\frac{n}{2}\int_0^{\pi} d\theta \sin\alpha \frac{d\alpha}{d\theta} \tag{3.10}$$

$$v_0 = \frac{1}{4}\left(\frac{dF}{dr}\right)^2 + \frac{\cos F(1-\cos F)}{r^2}\frac{1}{2}\int_0^{\pi} d\theta \sin\theta \left[\left(\frac{d\alpha}{d\theta}\right)^2 + n^2 \frac{\sin^2\alpha}{\sin^2\theta}\right] + \frac{F_\pi^2 m_\pi^2}{2F_K^2}(1-\cos F) \tag{3.11}$$

Diagonalization of the Hamiltonian, obtained from the Lagrangian (3.9), gives the following kaon eigenvalue equation (Dalarsson, (1993, 1995a–d, 1996a–c, 1997a–d):

$$\frac{d^2 k}{dr^2} + \frac{2}{r}\frac{dk}{dr} + [\omega^2 + 2\lambda(r)\omega - m_K^2 - v_0(r)]k = 0 \tag{3.12}$$

The hyperfine corrections to the dibaryon masses are obtained using the collective-coordinate quantization method, where we apply the time-dependent spatial (R) and isospin (A) rotations to the pion and kaon fields as follows:

$$u_\pi \rightarrow R A u_\pi A^{-1}, \qquad K \rightarrow RAK \tag{3.13}$$

The angular velocities in the body-fixed frame are given by

$$(R^{-1}\dot{R})_{ab} = \varepsilon_{abc}\Phi_c, \qquad A^{-1}\dot{A} = \frac{i}{2}\tau \cdot \Theta \tag{3.14}$$

Using a_1 and a_2 as coefficients of the up and down spinor χ, we obtain the rotational part of the total Lagrangian in the following form;

$$L_{\rm rot} = -\mathbf{T}\cdot\Theta + (1-c_1)[\tfrac{1}{2}(3\delta_{n,1} - 1)(T_1\Theta_1 + T_2\Theta_2) + T_3\Theta_3 - \delta_{n,1}(T_1\Phi_1 + T_2\Phi_2) - n(3\delta_{n,1} - 1)T_3\Phi_3] + \tfrac{1}{2}\Omega_1(\Phi_1^2 + \Phi_2^2) + \tfrac{1}{2}\Omega_2(\Theta_1^2 + \Theta_2^2 + \tfrac{1}{2}\Omega_3(n\Phi_3 - \Theta_3)^2 - \Omega_4\delta_{n,1}(\Phi_1\Theta_1 + \Phi_2\Theta_2) \tag{3.15}$$

where we used the definitions of the hyperfine constants (Dalarsson, 1993, 1995a–d, 1996a–c, 1997a–d)

$$c_1 = 1 - 2\omega_n \int_0^\pi d\theta \sin\theta \sin^2\alpha \int_\varepsilon^\infty r^2 dr\, k^*\, k \cos^2 \frac{F}{2} \tag{3.16}$$

$$c_2 = \frac{3}{2}(1 - \delta_{n,1}) + \frac{1}{2}(3\delta_{n,1} - 1)c_1 \tag{3.17}$$

In (3.17), ε is the constant cutoff defined as in (2.1), and we see that for $n = 1$ we have $c_1 = c_2 = c$, in agreement with Dalarsson (1993, 1995a–d, 1996a–c, 1997a–d). In (3.15), T^k is defined as $T^k = a_m \tau^k_{mn} a_n$, and Ω_k (k = 1, 2, 3, 4) are moments of inertia of the $SU(2)$ sector defined by

$$\Omega_1 = \frac{1}{2}\int_0^\pi d\theta \sin\theta \left[\left(\frac{d\alpha}{d\theta}\right)^2 + n^2 \frac{\sin^2\alpha}{\sin^2\theta}\right]\Omega_4 \tag{3.18}$$

$$\Omega_2 = \frac{3}{8}\int_0^\pi d\theta \sin\theta (1 + \cos^2\alpha)\Omega_4 \tag{3.19}$$

$$\Omega_3 = \frac{3}{4}\int_0^\pi d\theta \sin\theta \sin^2\alpha\, \Omega_4 \tag{3.20}$$

$$\Omega_4 = \Omega_{n=1} = \frac{2\pi}{3}\int_\varepsilon^\infty r^2\, dr \sin^2 F \tag{3.21}$$

From the results (3.18)–(3.20) it is easily seen that for $n = 1$, we obtain $\Omega_k = \Omega_4$ (k = 1, 2, 3). Furthermore, (3.21) is the well-known expression for the moment of inertia in the constant-cutoff model (Dalarsson, 1993, 1995a–d, 1996a–c, 1997a–d), where ε is the constant cutoff defined as in (2.1).

3.3. The Rotational Energies

The body-fixed components J_k^{bf} and I_k^{bf} of spin($\mathbf{J}$) and isospin($\mathbf{I}$), respectively, are defined by

$$J_k^{\mathrm{bf}} = \frac{\partial L}{\partial \Phi_k}, \qquad I_k^{\mathrm{bf}} = \frac{\partial L}{\partial \Theta_k} \tag{3.22}$$

The axial symmetry imposes the constraint $J_3^{\mathrm{bf}} = -n(I_3^{\mathrm{bf}} + T_3)$, and after the collective-coordinate quantization we obtain the rotational Hamiltonian in the form

$$H_{\mathrm{rot}} = \frac{1}{2\Omega_1}[\mathbf{J}^2 - (J_3^{\mathrm{bf}})^2] + \frac{1}{2\Omega_2}[\mathbf{I}^2 - (I_3^{\mathrm{bf}})^2] + \frac{c_2^2}{2\Omega_2}[\mathbf{T}^2 - T_3^2]$$
$$+ \frac{1}{2\Omega_3}(I_3^{\mathrm{bf}} + c_1 T_3)^2 + \frac{c_2}{2\Omega_2}(I_+^{\mathrm{bf}} T_- + I_-^{\mathrm{bf}} T_+) \quad (3.23)$$

The dibaryon states ($n = 2$), which satisfy all the constraints imposed by the symmetries of the system, are given by

$$|J, J_3; I, I_3; S\rangle = \frac{1}{8\pi^2}[2(1 + \delta_{I_3^{\mathrm{bf}},0}\delta_{T_3,0})]^{-1/2}\sqrt{(2J+1)(2I+1)}$$
$$\times [D^J_{J_3,-2k}(\Phi) D^I_{I_3,K-T_3}(\Theta) k_{T_3}(r, t) - (-1)^{I+J-S/2} D^J_{J_3,2K}(\Phi)$$
$$\times D^I_{I_3,-K+T_3}(\Phi) K_{-\mathrm{T}_3}(r, t)] \quad (3.24)$$

where S is the strangeness number and $K = I_3^{\mathrm{bf}} + T_3$. The parity of the state (3.24) is $(-1)^K$. It should be noted that the state (3.24) is not an eigenstate of the rotational Hamiltonian (3.23) since the last term in (3.23) does not commute with T_3. The proper eigenstates of the rotational Hamiltonian (3.23) will be the combinations of states (3.24) with the same $J, J_3; I, I_3; (J_3^{\mathrm{bf}})^2$ quantum numbers and different values of the T_3^2 quantum number.

For $S = 0$ and $S = -1$ only one value of T_3^2 is allowed (i.e., $T_3^2 = 0$ for $S = 0$ and T_3^2 and $= \frac{1}{4}$ for $S = -1$), and the eigenvalues of the rotation Hamiltonian (3.23) are given by

$$E_{\mathrm{rot}}^{s=0} = \frac{1}{2\Omega_1}[J(J+1) - (J_3^{\mathrm{bf}})^2] + \frac{1}{2\Omega_2}[I(I+1) - (I_3^{\mathrm{bf}})^2]$$
$$+ \frac{1}{2\Omega_3}(I_3^{\mathrm{bf}})^2 \quad (3.25)$$

and

$$E_{\mathrm{rot}}^{S=1} = \frac{1}{2\Omega_1}[J(J+1) - (J_3^{\mathrm{bf}})^2] + \frac{1}{2\Omega_3}\left\{(1 - c_1)\left[(I_3^{\mathrm{bf}})^2 - \frac{c_1}{4}\right] + \frac{c_1}{4}(J_3^{\mathrm{bf}})^2\right\}$$
$$+ \frac{1}{2\Omega_2}\left[I(I+1) - (I_3^{\mathrm{bf}})^2 + c_2\left(\frac{c_2}{2} - (-1)^{I+J+1/2}\delta_{J_3,0}\sqrt{I(I+1) + \frac{1}{4}}\right)\right] \quad (3.26)$$

For $S = -2$, two values of T_3^2 (i.e. $T_3^2 = 0$ and $T_3^2 = 1$) are in general allowed, and E_{rot} is given by the eigenvalues of

$$E_{\mathrm{rot}}^{S=0} = \frac{1}{2\Omega_1}\left[J(J+1) - (J_3^{\mathrm{bf}})^2\right] + \frac{1}{2\Omega_2}\left[I(I+1) - c_2^2\right] + \frac{1}{2\Omega_3}\, c_1 K^2$$
$$+ \frac{c}{2\Omega}\begin{bmatrix} \dfrac{c_2^2 - K^2}{2\Omega_2} + \dfrac{1 - c_1}{2\Omega_3} K^2 & \dfrac{c_2}{\sqrt{2}\,\Omega_2}\sqrt{1 + \delta_{K,\mathrm{O}}}\sqrt{(I \mp K)(I \pm M)} \\ \dfrac{c_2}{\sqrt{2}\,\Omega_2}\sqrt{1 + \delta_{K,\mathrm{O}}}\sqrt{(I \mp K)(I \pm M)} & -\dfrac{M^2}{2\Omega_2} + \dfrac{1 - c_1}{2\Omega_3}(M^2 - c_1) \end{bmatrix} \tag{3.27}$$

where $M = K \pm 1$. The 2×2 matrix is constructed using (3.24) ordered according to increasing T_3^2.

3.4. Numerical Results

In the numerical calculations we use the empirical values of all the parameters F_π, m_π, F_K, m_K and the calculated value of the constant-cutoff ε given by (2.14) for $F_\pi = 186$ MeV. The results of the present calculations, compared to those obtained using the complete Skyrme model in [13], are given in Table 1.

As in the case of the complete Skyrme model [13], we use only g_1 as the variational parameter and assume that $g_k = 0$ for $k \neq 1$ in the variational expansion (3.6), when calculating the minimum in the $B = 2$ soliton mass. The inclusion of g_2 as a second variational parameter does not lead to any significant improvement in the energy minimum of the $\mathrm{B} = 2$ soliton.

From Table I we see that there is a qualitative agreement between the present results and results obtained using the complete Skyrme model (Thomas *et al.*, 1994). The results are in general somewhat closer to those obtained in Thomas *et al.* (1994) using the parameter set *A*, although there is no correlation to any of the two sets used there.

If we now turn to the problem of the H-particle stability, we obtain in the present model $M(\mathrm{H}) - 2M(\Lambda) = -41$ MeV, indicating a binding which is only slightly stronger than the one obtained in the complete Skyrme model (Thomas *et al.*, 1994), i.e., $M(\mathrm{H}) - 2M(\Lambda) = -34$ MeV. As argued in Thomas *et al.* (1994), the dynamical departures from the lowest energy solution, possibly parametrized as zero-point fluctuations of a conveniently chosen collective coordinate, may decrease the binding energies. Another

Table I. Rotational Contributions to the Dibaryon States (B=2) in MeV[a]

	(I, J^{π})	E_{rot}	E_{rot} (Thomas *et al.*, 1991) Set A	Set B
$S = 0$	$0, 1^+$	71	75	61
	$1, 0^+$	131	113	93
	NN	152	147	147
	$1, 2^-$	201	216	177
$S = -1$	$\frac{1}{2}, 0^+$	65	61	46
	$\frac{1}{2}, 1^+$	81	87	75
	$N\Lambda$	97	92	85
	$\frac{3}{2}, 0^+$	169	157	138
	$\frac{1}{2}, 2^-$	81	87	75
$S = -2$	$0,0^+$	37	21	11
	$\Lambda\Lambda$	31	37	22
	$1, 0^+$	91	79	66
	$1, 1^+$	111	107	98
	$0, 2^-$	137	119	88
	$1, 2^-$	175	187	152
	$0, 2^+$	221	247	195

[a] NN, NΛ, and $\Lambda\Lambda$ are sums of the rotational contributions to the corresponding particles, serving as rotational thresholds in each particular group of states.

effect that may decrease the binding energies is the Casimir effect leading to the $\mathcal{O}(N_C^0)$ contributions. Due to the relatively weak binding obtained both in the complete Skyrme model (Thomas *et al.*, 1994) and here, it is likely that the above two mechanisms may be strong enough to unbind the H-particle. A more datailed account of these effects can be found in Thomas *et al.* (1994) and in references therein.

4. CONCLUSIONS

The present paper shows the possibility of using the Skyrme model for calculation of the rotational energies and spectra of axially symmetric dibaryons without using the Skyrme stabilizing term, proportional to e^{-2}, which makes both the analytic and numerical treatment more difficult.

For such a simple model with only one arbitrary dimensional constant, and where all parameters (F_{π}, m_{π}, F_K, m_K) are chosen equal to their empirical values, there is qualitative agreement of the results for the rotational contributions to the axially symmetric dibaryon masses with the corresponding predictions of the complete Skyrme model (Thomas *et al.*, 1994).

Furthermore, we find that in the present approach, similarly to the case of the complete Skyrme model (Thomas *et al.*, 1994), the H-particle is bound, even though the neglected vacuum effects might contribute to the unbinding of this particle.

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